

## Children's solving of mathematical word problems: The contribution of working memory

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The aim of this study was to examine the contribution of working memory to children's solving of change and compare word problems. First- to fourth-grade primary school students ( $N=283$ ) participated in the study. Two working memory measures were used: backwards digit span and listening span. On the whole sample, both predictors significantly explained the achievement variance in compare problems. For change problems, the results were different for younger and older students. For first and second grade students the achievement in listening span task was a significant predictor of the achievement in change problems, while the achievement in backward digit span task was not a significant predictor. However, for third and fourth grade students, none of the predictors were significant. The results confirmed the assumptions of theories which emphasize the importance of linguistic factors in solving mathematical word problems.

*Key words:* change problems, compare problems, working memory span, primary school students

Learning mathematics includes solving various types of problems, from those which require performing arithmetical operations to those which require problem solving skills. Children are faced with mathematical word problems consisting of both words and numbers as early as the preschool age, and also later in school mathematics. Effective solving of these problems primarily requires comprehension.

The classic classification of addition and subtraction word problems is the one put forward by Heller and Greeno (1978), which was revised and expanded by enlarging the number of problems by Riley and Greeno (1988). The first criterion for the classification of word problems is the semantic relationship describing the problem situation: combining, increasing, decreasing or comparing sets of objects. Thus, with regard to this criterion there are combine, change and compare problems. An example of the combine problem is: "Mary has two apples. Jane has six apples. How many apples do they have together?". An example of the change problem is "Mary had seven apples. Then Jane gave her two apples. How many apples does Mary have now?" and an example of the compare problem is: "Mary has three apples. Jane has five apples. How many apples does Jane

have more than Mary?". The second classification criterion is the position of the unknown quantity. According to this criterion, each of the abovementioned three categories can be further divided into six types of problems, which means that there is a total of eighteen types of problems. A detailed table representation of this classification can be found in Riley and Greeno (1988).

Results of previous studies indicate that compare problems are the most difficult for children (Riley & Greeno, 1988; Riley, Greeno, & Heller, 1983). Some studies have found that combine problems are more difficult than change problems for preschool children and first-graders (Nesher & Katriel, 1978; Vergnaud, 1982), while other studies have found no significant differences in the difficulty of these two types of problems (Cummins, Kintsch, Reusser, & Weimer, 1988; Riley & Greeno, 1988). On a sample of preschool children and first-, second- and third-grade primary school students, Riley and Greeno (1988) have found that the proportion of correct solutions in all types of problems increases with age, but that the relative difficulty of problem types and particular problems remains the same.

Models that explain how children solve mathematical word problems differ according to their assumptions about the development of children's capacities which improve their achievement in mathematics. According to mathematical-logical models, solving word problems primarily requires conceptual knowledge (Briars & Larkin, 1984; Riley, Greeno, & Heller, 1983). Linguistic models emphasize the importance of understanding and interpreting the text of the word problem (DeCorte & Verschaffel, 1985; Kintsch &

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Greeno, 1985; Nathan, Kintsch, & Young, 1992; Reusser, 1989). Thus, Kintsch and Greeno (1985) believe that solving a mathematical word problem begins with text comprehension. On the basis of the textual form of the problem, a text representation is created, which is the basis for solving the problem mathematically. The text base contains the basic data from the text of the problem, and the model of the problem contains the relevant data from the text in a form suitable for performing the arithmetic operation. The solver constructs the model of the problem by inferring which data is required in solving the problem and is not included in the text base, while excluding the data which is unnecessary, but is part of the text base.

Reusser's (1989) SPS model (Situation Problem Solver) is also based on the assumptions of linguistic models. Reusser believes that there are four sources of difficulties in solving mathematical word problems: the verbal formulation of the problem text, the situational context (events and relationships between characters in the background of the text), the conceptual logico-mathematical or arithmetical knowledge about set relations and arithmetical skills required for counting or solving equations. According to Reusser (1989), understanding word problems involves the interplay between linguistic knowledge, being familiar with possible real-world situations and mathematical knowledge. Children's major difficulties in solving mathematical word problems are a result of insufficient understanding of linguistic structures and situations given in the problem, while a lack of logico-mathematical knowledge and of arithmetical skills is of minor importance.

Thus, mathematical word problem solving includes monitoring and coordinating multiple processes, such as reading, language comprehension, problem representation, selection and execution of calculation operations (Kintsch & Greeno, 1985; Mayer & Hegarty, 1996; Swanson, 2004). These processes require working memory capacity. As distinguished from short-term memory, which is responsible for temporary information storage, working memory refers to the capacity to store information over brief periods of time while simultaneously processing the same or other information (Baddeley & Logie, 1999; Miyake, 2001). One of the most prominent working memory models, which is frequently used in studies dealing with mathematical cognition, is Baddeley's model (Baddeley, 1986, 1996). According to this model there are two components of the working memory: the central executive and two slave systems, the articulatory loop and the visual-spatial sketchpad. The articulatory loop is responsible for temporarily storing verbal information, which is maintained by rehearsal. The visual-spatial sketchpad is responsible for temporary storage of visual-spatial information and is crucial in creating mental images and their manipulation. Both slave systems are in direct contact with the central executive, which is responsible for their coordination and supervising as well as for selective attention. Baddeley (2000) also suggested a fourth

component of the model, the episodic buffer, which integrates and temporarily stores information from the articulatory loop and the visual-spatial sketchpad and enables the exchange of information between the central executive and long-term memory.

As opposed to Baddeley's model, according to which the working memory consists of several components, some authors believe that it is a global function, and that inter-individual differences in the working memory capacity reflect the capacity of the central executive (Cowan, 1999; Daneman & Carpenter, 1980; Engle, Tuholski, Laughlin, & Conway, 1999a).

Regardless of these differences between the models, researchers generally agree that working memory is particularly important in activities which include complex cognitive processes, such as text comprehension and solving various mathematical problems (Daneman & Carpenter, 1980; Hitch, Towse, & Hutton, 2001). For example, DeBeni, Palladino, Pazzaglia, and Cornoldi (1998) have found that children with text comprehension difficulties have a smaller working memory capacity, measured using Daneman and Carpenter's test (Listening Span Test, 1980). In this test, participants listen to sets of increasing number of sentences, and are required to recall the last word from each sentence in the correct sequence. Numerous studies have also shown that mathematical difficulties are connected with poor working memory (e.g. Ostad, 1998; Siegel & Ryan, 1989; Swanson, 1993). For example, Geary, Hoard, Byrd-Craven, and DeSoto (2004) have shown that first-graders with difficulties in mathematics have a smaller numerical working memory capacity and are less successful in solving addition problems. Passolunghi and Siegel (2001) have found that poor mathematical problem solvers do worse on verbal and numerical measures of working memory and have difficulties in inhibiting irrelevant information.

Although numerous studies examine the contribution of working memory to solving written mathematical calculation tasks, a few studies have focused on the contribution of working memory to mathematical word problem solving (Andersson, 2007; Kail & Hall, 1999; Lee, Ng, Ng, & Lim, 2004; Swanson, 2004, 2006; Swanson & Beebe-Frankenberg, 2004; Swanson & Sachse-Lee, 2001). These studies show that working memory tasks requiring concurrent processing and storage of verbal or visual information are significant predictors of mathematical word problem solving aside from phonological processing, reading ability, skills in mathematical calculation and fluid IQ.

The aim of this study was to examine the contribution of working memory to children's solving of change and compare word problems. Given that compare problems are linguistically more complex than change problems, we assumed that the contribution of working memory in solving the former will be greater than in solving the latter. We also assumed that the contribution of working memory in word problem solving would be greater in younger than in older

children. Younger children have smaller working memory span than older children (e.g. Gathercole, Pickering, Adair, & Wearing, 2004) and we can assume that because of that in first and second grade working memory would be more important for mathematical word problem solving than in later grades. For example, among typically developing third and fourth graders, Swanson, Cooney, and Brock (1993) found only a weak relation between working memory and problem solution accuracy, and this relation disappeared once reading comprehension was considered.

We used two measures of working memory in the study: backward digit span and listening span. In order to solve a mathematical word problem, a child has to integrate in working memory information on numbers and relations mentioned in the problem into a problem representation. Although words and numbers are both verbal materials, we were interested to see whether the achievement in one of these tasks will be more related to achievement in mathematical word problems.

## METHOD

### *Participants*

A total of 283 students (160 boys and 123 girls) from two primary schools in Zagreb, Croatia, participated in the study. Out of this, 49 were first-grade primary school students ( $M = 7.7$  years;  $SD = 0.28$ ), 83 were second-grade students ( $M = 8.6$  years;  $SD = 0.40$ ), 78 were third-grade students ( $M = 9.6$  years;  $SD = 0.35$ ) and 73 were fourth-grade students ( $M = 10.6$  years;  $SD = 0.36$ ). All of the students attended regular school curriculum and they did not have any specific learning difficulties in math or language.

### *Tasks*

Working memory span protocols, mathematical word problems protocols, answer sheets and cassette recorders were used in the study.

**Backward digit span task.** The task is an adaptation of the WISC-R backward digit span subtest (Wechsler, 1974). Although recently there were some disagreements regarding backward digit span as a measure of working memory (some researchers treat it as a measure of short term memory, e.g. Engle, Tuholski, Laughlin, & Conway, 1999b), it is widely used, because both storage and manipulation of information are needed to reverse the order of digits during recall. Some span tasks include dual task paradigm, in which subjects have to perform secondary task that ensures the additional work load and leads to recruitment of working memory capacity (e.g. Olive, 2004). However, due to the age of our participants, we decided not to use such a demanding task.

The experimenter read a series of digits to the child, and the child had to verbally recall them in the reverse order of presentation. The shortest span consisted of two digits, and the longest of eight. Numbers were read at a rate of one digit per second. The test was preceded by two practice trials consisting of two shortest digit spans (2 digits). The length of the span was gradually increased, and there were two series of digits for each span. Testing stopped when the child made a mistake in both trials of the same span length. Each correctly recalled series in reverse order was scored as one point, and each incorrectly recalled series as zero points. The maximum score that a child could get in this test was 14. Cronbach's alpha for backward digit span in this study was .73.

**Listening span task.** The task is an adaptation of the task devised originally by Daneman and Carpenter (1980). The examiner read to the child a series of short sentences (the sentences consisted of 4 to 5 well-known words). The shortest series consisted of two sentences and the longest of six. The number of sentences in the series was gradually increased, and there were two sets for a particular series of sentences. The child was asked to say the final word of each sentence after each presented series in the correct sequence. For instance, if the following two sentences were read to the child: "Children are climbing a hill" (in original: "Djeca se penju na brdo") and "The giraffe has a long neck" (in original: "Žirafa ima dugačak vrat"), the child was supposed to respond "Hill, neck" ("Brdo, vrat"). The test was preceded by two practice trials consisting of the shortest series of sentences (i.e. 2 sentences). Testing stopped when the child failed to recall final words in both sets of a particular series of sentences (for instance in both sets of 4 sentences) in the correct sequence. One point was assigned for each correctly recalled sequence of final words from a particular series. The maximum score that a child could get in this test was 10. Cronbach's alpha was .76.

**Mathematical word problems task.** Mathematical word problems protocols consisted of 32 word problems (16 change problems and 16 compare problems) and one example (practice trial). The sequence of the problems in the protocol was rotated in three random orders. Eight change problems Type 3 were used in the study (for example: "Marija had three balloons. Then she got several balloons from Vlasta. Now Marija has eight balloons. How many balloons did Marija get from Vlasta?") and eight change problems Type 6 (for example: "Bruno had several marbles. Then Ana got five marbles from Bruno. Now Bruno has four marbles. How many marbles did Bruno start with?"). As for compare problems, there were eight Type 3 problems (for instance: "Tea has six cookies. Ines has three cookies more than Tea. How many cookies does Ines have?") and eight Type 5 problems (for example: "Matej has eight crayons. He has two crayons more than Denis. How many crayons does Denis have?"). Combine problem Type 1 was used as the practice trial (e.g. "Bojan has five apples. Sanja has three

apples. How many apples do they have together?"). Two versions of the problems were used, and they differed in the numbers used, but not in the text. The first version consisted of addition and subtraction of numbers from 2 to 9, with the result within the same interval. This version was used with first- and second-grade students. Since these problems are too easy for third- and fourth-graders, they were given the second version, which required addition and subtraction of numbers between 3 and 29, and the result was within the 20 to 29 interval. In both versions, problems were constructed in such a way that the result was never a number used in the problem (for instance, the combination  $6 - 3 = 3$  was not used). One point was scored for each correct solution, so the maximum score that a child could get for change problems and for compare problems was 16. Cronbach's alpha for change problems subset was .82. Cronbach's alpha for compare problems subset was .84.

### Procedure

Parental consent was obtained for children's participation in the study. Data were collected by 16 specially trained psychology undergraduate students in five successive days. Children were tested individually in a separate room, and the testing, depending on the child, lasted between 15 and 25 minutes. Working memory was tested first, followed by the testing of mathematical word problems. In half of the children, backward span task was administered first, and for the other half listening span task was administered first. In the mathematical word problems task, the experimenter read the problem to the child, and the child was asked to calculate the answer in the head and explain how s/he obtained

it. Children's answers and their explanations about how they solved the problem were recorded in answer sheets and using tape recorders in order to analyze problem solving strategies. These analyses are not included in this paper.

## RESULTS

Means and standard deviations of the results in the change and compare problems for first- to fourth-grade students are shown in Table 1.

Mixed-model ANOVA was used to check whether there was a difference with regard to grade and mathematical word problem type. The main effect of grade was significant ( $F(3,279) = 20.34$ ;  $p < .001$ ), however, the differences are not very large (Partial Eta Squared = .18). Scheffé post-hoc tests showed that second grade students were significantly better than the first grade students ( $p < .01$ ), third grade students were better than second grade students ( $p < .04$ ), while there were no differences between the third and the fourth grade ( $p = .62$ ).

The main effect of type of mathematical word problem was also significant ( $F(1,279) = 8.92$ ;  $p < .01$ ). This is a small difference (Partial Eta Squared = .03). Achievement in change problems was significantly better than achievement in compare problems.

Interaction between problem type and grade was also significant ( $F(3,279) = 2.93$ ;  $p < .04$ ). Partial Eta Squared was .03. Dependent samples t-tests showed that achievement in change problems was better than the achievement in compare problems in the first grade ( $t(48) = 2.02$ ;  $p < .05$ ), in the second grade ( $t(82) = 2.72$ ;  $p < .01$ ) and in the third grade

Table 1

Means and standard deviations of results in change and compare problems for the four grades, and overall results for each type of problem and age group

|                  | 1 <sup>st</sup> grade<br>( <i>n</i> =49) |           | 2 <sup>nd</sup> grade<br>( <i>n</i> =83) |           | 3 <sup>rd</sup> grade<br>( <i>n</i> =78) |           | 4 <sup>th</sup> grade<br>( <i>n</i> =73) |           | Total<br>( <i>N</i> =283) |           |
|------------------|--|-----------|--|-----------|--|-----------|--|-----------|---------------------------|-----------|
|                  | <i>M</i>                                 | <i>SD</i> | <i>M</i>                                 | <i>SD</i> | <i>M</i>                                 | <i>SD</i> | <i>M</i>                                 | <i>SD</i> | <i>M</i>                  | <i>SD</i> |
| Change problems  | 10.39                                    | 3.90      | 12.27                                    | 3.76      | 13.38                                    | 2.68      | 13.52                                    | 2.39      | 12.57                     | 3.38      |
| Compare problems | 9.51                                     | 3.62      | 11.22                                    | 3.72      | 12.76                                    | 3.02      | 13.85                                    | 2.40      | 12.02                     | 3.54      |
| All problems     | 19.90                                    | 6.72      | 23.48                                    | 6.60      | 26.14                                    | 4.96      | 27.37                                    | 4.08      | 24.60                     | 6.18      |

Table 2

Means and standard deviations of results in backward digit span task and listening span task for the four grades

|                          | 1 <sup>st</sup> grade<br>( <i>n</i> =49) |           | 2 <sup>nd</sup> grade<br>( <i>n</i> =84) |           | 3 <sup>rd</sup> grade<br>( <i>n</i> =78) |           | 4 <sup>th</sup> grade<br>( <i>n</i> =73) |           | Total<br>( <i>N</i> =283) |           |
|--------------------------|--|-----------|--|-----------|--|-----------|--|-----------|---------------------------|-----------|
|                          | <i>M</i>                                 | <i>SD</i> | <i>M</i>                                 | <i>SD</i> | <i>M</i>                                 | <i>SD</i> | <i>M</i>                                 | <i>SD</i> | <i>M</i>                  | <i>SD</i> |
| Backward digit span task | 3.49                                     | 1.21      | 4.30                                     | 1.47      | 5.06                                     | 1.66      | 5.21                                     | 1.68      | 4.60                      | 1.66      |
| Listening span task      | 2.49                                     | 1.53      | 3.51                                     | 1.56      | 4.64                                     | 1.53      | 4.92                                     | 1.55      | 4.01                      | 1.77      |



( $t(77) = 1.99$ ;  $p=.05$ ), while there was no such difference between the achievement in change and compare problems in the fourth grade ( $t(72) = 1.12$ ;  $p=.27$ ).

In previous studies (Delgado & Prieto, 2004; Hyde, Fennema & Lamon, 1990) no gender differences have been found in solving mathematical word problems in children under the age of 12. Similarly, no significant differences between boys and girls in mathematical word problem achievement were found in this study ( $F(1,281) = 0.03$ ;  $p = .87$ ; and  $F(1,281) = 0.23$ ;  $p=.64$  for change problems and compare problems, respectively).

Table 2 shows means and standard deviations of achievement in two working memory tasks for students from grades one through four.

We checked for differences between different grades for the two measures of working memory span. For the backward digit span task, one-way ANOVA revealed significant differences between different grades ( $F(3,280) = 15.63$ ;  $p<.001$ ), however the effect size is not very large (Partial Eta Squared = .14). Scheffé post-hoc tests showed that second grade students were significantly better than the first grade students ( $p<.04$ ), third grade students were better than second grade students ( $p<.02$ ), while there were no differences between the third and the fourth grade ( $p=.96$ ). Similar results were obtained for the listening span task: one-way ANOVA revealed significant differences between the grades ( $F(3,280) = 31.44$ ;  $p<.001$ ). Partial Eta Squared was .25. Scheffé post-hoc tests showed that second grade students were significantly better than the first grade students ( $p<.01$ ), third grade students were better than second grade students ( $p<.001$ ), while there were no differences between the third and the fourth grade ( $p=.75$ ).

One-way ANOVA showed that there were no differences between girls and boys with regard to their working memory span ( $F(1,282) = 1.88$ ;  $p=.17$  and  $F(1,282) = 2.44$ ;  $p=.12$  for backward digit span task and listening span task, respectively).

As a first step to explore the contribution of working memory to children's mathematical word problem solving, correlations were calculated among all tasks used in the study. The results are presented in Table 3. All correlation coefficients were significant.

Table 3  
Correlations between the tasks used in the study ( $N=283$ )

| Tasks               | Compare problems | Backward digit span | Listening span |
|---------------------|------------------|---------------------|----------------|
| Change problems     | .60**            | .28**               | .40**          |
| Compare problems    | —                | .30**               | .44**          |
| Backward digit span |                  | —                   | .39**          |
| Listening span      |                  |                     | —              |

Note. \*\* $p<.01$ .

In order to examine whether there are interaction effects between grade and working memory span, we conducted two hierarchical regression analyses, with results in change problems and compare problems as dependent variables. In the regression on compare problems, we entered grade, backward digit span and listening span as block one. Grade by backward digit span interaction and grade by listening span interaction terms were entered as block two.  $R^2$  for the full model was .27,  $p<.001$ . Variables entered as block one significantly predicted results in compare problems ( $R^2=.26$ ;  $F(3,279) = 32.77$ ;  $p<.001$ ). There was no significant contribution of the grade and working memory measures interaction terms ( $\Delta R^2 = .01$ ;  $F(2,277) = 2.51$ ;  $p=.08$ ). Thus, we conducted multiple regression analysis with results in working memory measures as predictors and results in compare

Table 4  
Results of the multiple regression analysis for the criterion variable of achievement in compare problems for all participants ( $N = 283$ )

| Criterion variable              | Predictors               | $\beta$ | $r_s$ | $R$  | $R^2$  |
|---------------------------------|--------------------------|---------|-------|------|--------|
| Achievement in compare problems | Backward digit span task | .145*   | .132  | .461 | .212** |
|                                 | Listening span task      | .382**  | .348  |      |        |

Note.  $\beta$  – standardized partial regression coefficient;  $r_s$  – semi-partial correlation coefficient;  $R$  – multiple correlation coefficient;  $R^2$  – multiple determination coefficient; \* $p<.05$ ; \*\* $p<.01$

Table 5  
Results of multiple regression analysis for the criterion variables of achievement in change problems for younger students (1<sup>st</sup> and 2<sup>nd</sup> grade;  $n = 132$ )

| Criterion variable             | Predictors               | $\beta$ | $r_s$ | $R$  | $R^2$  |
|--------------------------------|--------------------------|---------|-------|------|--------|
| Achievement in change problems | Backward digit span task | .107    | .113  | .463 | .214** |
|                                | Listening span task      | .417**  | .406  |      |        |

Note.  $\beta$  – standardized partial regression coefficient;  $r_s$  – semi-partial correlation coefficient;  $R$  – multiple correlation coefficient;  $R^2$  – multiple determination coefficient; \*\* $p<.01$

Table 6  
Results of multiple regression analysis for the criterion variables of achievement in change problems for older students (3<sup>rd</sup> and 4<sup>th</sup> grade;  $n = 151$ )

| Criterion variable             | Predictors               | $\beta$ | $r_s$ | $R$  | $R^2$ |
|--------------------------------|--------------------------|---------|-------|------|-------|
| Achievement in change problems | Backward digit span task | .145    | .141  | .206 | .043* |
|                                | Listening span task      | .111    | .109  |      |       |

Note.  $\beta$  – standardized partial regression coefficient;  $r_s$  – semi-partial correlation coefficient;  $R$  – multiple correlation coefficient;  $R^2$  – multiple determination coefficient; \* $p<.05$ .

problems as dependent variable for all participants, regardless of grade. The results are presented in Table 4. The table shows that backward digit span and listening span are both significant predictors, which account for 21.2 % of achievement variance in compare problems.

In the hierarchical regression analysis on change problems, we also entered grade, backward digit span and listening span as block one, and grade by backward digit span interaction and grade by listening span interaction as block two.  $R^2$  for the full model was .21,  $p < .001$ . Variables entered as block one significantly predicted results in change problems ( $R^2 = .19$ ;  $F(3,279) = 32.77$ ;  $p < .001$ ), and the contribution by the second block, i.e. the grade and working memory measures interaction terms, was also significant ( $\Delta R^2 = .03$ ;  $F(2,277) = 4.41$ ;  $p < .02$ ). Because of this interaction, we conducted multiple regression analyses with working memory measures as predictors and results in change problems as dependent variable separately for younger (first and second grade) students and for older (third and fourth grade) students. Table 5 shows the results of the multiple regression analysis for younger students.

The achievement in listening span task was a significant predictor of the achievement in change problems, while the achievement in backward digit span task was not a significant predictor. Table 6 shows the results of multiple regression analysis for older students. Multiple determination coefficient is very small, although significant, however none of the predictors were significant.

## DISCUSSION

### *Differences with regard to grade and mathematical word problem type*

The results show that the achievement in change and compare problems increases with grade and that compare problems are harder than change problems for the students in first, second and third grade. These results are in accordance with the results of other studies, which have shown that compare problems are the most difficult type of mathematical word problems (Riley & Greeno, 1988; Vlahović-Štetić, Kišak, & Vizek-Vidović, 2000; Vlahović-Štetić, Rován, & Mendek, 2004). Previous research has also shown that mathematical word problem achievement increases with age (DeCorte, Verschaffel, & De Win, 1985; Riley & Greeno, 1988; Vlahović-Štetić, Rován & Mendek, 2004). Of course, it is to be expected that older children should calculate better, despite the fact that they have to deal with larger numbers, and it is also expected that their mathematical and logical knowledge is more extensive. However, in accordance with linguistic models, it can be assumed that children in different age groups differ according to their ability to understand the text and the situation given in the problem, and that the

understanding is better in older children, which leads to better mathematical word problem solving.

### *Differences with regard to grade and working memory span measure*

The achievement in working memory tasks increases with grade, which is in accordance with findings from other studies (Kail, 1990; Gathercole et al., 2004). Second grade students' achievement in both tasks is better than that of the first grade students, while the third grade students are more successful than those of the second grade. However, between the third and fourth grade students there is no difference in the backward digit span task or in the listening span task. Further investigations with children aged 11 to 15 years should clarify developmental changes in these tasks.

### *Working memory and children's mathematical problem solving*

The main aim of the present study was to explore the contribution of working memory to children's solving of change and compare word problems. We used two measures of working memory in the study: backward digit span and listening span. The correlation between these two measures was .39. Although these both measures are verbal materials, they are obviously not the same, and it is justified to use both of them in the examination of the contribution of working memory in mathematical word problem solving.

Hierarchical regression analysis showed that for the compare problems results of all students lie on the same regression line, so we conducted multiple regression analysis with two working memory measures as predictors and results in compare problems as a criterion for all students, regardless of the grade. However, for the change problems, hierarchical regression analysis showed that the results for all grades do not lie along the same regression line, so the results could not be analysed for all participants together. For the clarity of interpretation, we conducted multiple regression analyses separately for younger students (1<sup>st</sup> and 2<sup>nd</sup> grade) and for older students (3<sup>rd</sup> and 4<sup>th</sup> grade). The results for compare problems showed that backward digit span task and listening span task together account for 21.2% of variance of achievement in compare problems. For change problems, for the younger students these two measures account for 21.4% of achievement variance. However for the older students they account only for 4.3% of achievement variance. This is partly in accordance with our hypotheses. Compare problems are linguistically more complex than change problems, so the contribution of working memory in solving them is important in all grades, while the contribution of working memory in solving change problems is important only in earlier grades. It can be assumed that in third and fourth grade, and in older children, working memory

span probably becomes less significant for the achievement in change problems, but some other factors (such as understanding the text and the situation, understanding mathematical and logical relations, arithmetic skills) still remain important. Swanson et al. (1993) found only a weak relation between working memory and problem solution accuracy among third and fourth graders, and this correlation disappeared once reading comprehension was considered.

For the achievement in compare problems and for the achievement in change problems (in first two grades) listening span task accounts for a larger percentage of criterion variance than backward digit span task. This is in accordance with linguistic models, which assume that word problems require understanding of the text, and not only arithmetical skills. In order to solve a mathematical word problem, text manipulation is required, so it is to be expected that working memory capacity should have an effect on children's achievement. Kintsch and Greeno's (1985) model, thus, apart from analyses within the mathematical-logical models also takes into consideration the model of text comprehension developed by Kintsch and Van Dijk (1978). According to Kintsch's more recent model of reading comprehension (Kintsch, 1998), a number of text propositions are kept simultaneously in the working memory, which enables their integration. Similarly, during comprehension of a mathematical word problem, statements necessary for solving mathematical word problems are integrated in the working memory (Swanson, 2004). Daneman and Carpenter (1980) have found that working memory capacity measure like the one that was used in this research significantly correlates with different reading comprehension measures, whereas they have found no correlation between digit span and text understanding.

In addition to establishing the contribution of working memory capacity, practical implications of these results are also a significant issue. Our results suggest that in younger children textual comprehension and verbal working memory capacity are important in solving mathematical word problems. Obviously, while teaching, understanding of the problems should be checked and practiced, for instance, children may be asked to retell what was asked in the problem in their own words. In this way we can ensure that children acquire the knowledge of various linguistic structures that are used in mathematical problems.

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